Oxford Cambridge and RSA

Thursday 23 June 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Three sequences, a_n, b_n and c_n, are defined for $n \geqslant 1$ by the following recurrence relations.

$$(a_{n+1} - 2)(2 - a_n) = 3 \text{ with } a_1 = 3$$

$$b_{n+1} = -\tfrac{1}{2}b_n + 3 \text{ with } b_1 = 1.5$$

$$c_{n+1} - \frac{c_n^2}{n} = 1 \text{ with } c_1 = 2.5$$

The output from a spreadsheet which presents the first 10 terms of a_n, b_n and c_n, is shown below.

	A	B	C	D
1	n	a_n	b_n	c_n
2	1	3	1.5	2.5
3	2	–1	2.25	7.25
4	3	3	1.875	27.28125
5	4	–1	2.0625	249.0889
6	5	3	1.96875	15512.32
7	6	–1	2.01563	48126390
8	7	3	1.99219	3.86E+14
9	8	–1	2.00391	2.13E+28
10	9	3	1.99805	5.66E+55
11	10	–1	2.00098	3.6E+110

Without attempting to solve any recurrence relations, describe the apparent behaviour, including as $n \to \infty$, of

- a_n
- b_n
- c_n **[7]**

2 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 10 & 12 & -8 \\ -1 & 2 & 4 \\ 3 & 6 & 2 \end{pmatrix}$.

(a) **In this question you must show detailed reasoning.**

Show that the characteristic equation of **A** is $-\lambda^3 + 14\lambda^2 - 56\lambda + 64 = 0$. **[3]**

(b) Use the Cayley-Hamilton theorem to determine $\mathbf{A}^{-1}$. **[5]**

A matrix **E** and a diagonal matrix **D** are such that $\mathbf{A} = \mathbf{EDE}^{-1}$. The elements in the diagonal of **D** increase from top left to bottom right.

(c) Determine the matrix **D**. **[4]**

3 A sequence is defined by the recurrence relation $5t_{n+1} - 4t_n = 3n^2 + 28n + 6$, for $n \geqslant 0$, with $t_0 = 7$.

(a) Find an expression for t_n in terms of n. **[6]**

Another sequence is defined by $v_n = \dfrac{t_n}{n^m}$, for $n \geqslant 1$, where m is a constant.

(b) In each of the following cases determine $\lim\limits_{n \to \infty} v_n$, if it exists, or show that the sequence is divergent.

(i) $m = 3$ **[1]**

(ii) $m = 2$ **[1]**

(iii) $m = 1$ **[1]**

4 A binary operation, $\circ$, is defined on a set of numbers, A, in the following way.

$$a \circ b = k_1 a - k_2 b + k_3, \text{ for } a, b \in A,$$

where k_1, k_2 and k_3 are constants (which are not necessarily in A) and the operations addition, subtraction and multiplication of numbers have their usual notation and meaning.

You are initially given the following information about $\circ$ and A.

- $A = \mathbb{R}$
- $0 \circ 0 = 2$
- An identity element, e, exists for $\circ$ in A

(a) Show that $a \circ b = a + b + 2$. **[5]**

(b) State the value of e. **[1]**

(c) Explain whether $\circ$ is commutative over A. **[1]**

(d) Determine whether or not $(A, \circ)$ is a group. **[6]**

(e) Explain whether your answer to part **(d)** would change in each of the following cases, giving details of any change.

(i) $A = \mathbb{Z}$ **[1]**

(ii) $A = \{2m : m \in \mathbb{Z}\}$ **[1]**

(iii) $A = \{n : n \in \mathbb{Z}, n \geqslant -2\}$ **[1]**

5 A surface S is defined by $z = \mathrm{f}(x, y)$, where $\mathrm{f}(x, y) = y\mathrm{e}^{-(x^2+2x+2)y}$.

(a) **(i)** Find $\dfrac{\partial \mathrm{f}}{\partial x}$. **[1]**

(ii) Show that $\dfrac{\partial \mathrm{f}}{\partial y} = -(x^2y + 2xy + 2y - 1)\mathrm{e}^{-(x^2+2x+2)y}$. **[1]**

(iii) Determine the coordinates of any stationary points on S. **[4]**

Fig. 5.1 shows the graph of $z = \mathrm{e}^{-x^2}$ and **Fig. 5.2** shows the contour of S defined by $z = 0.25$.

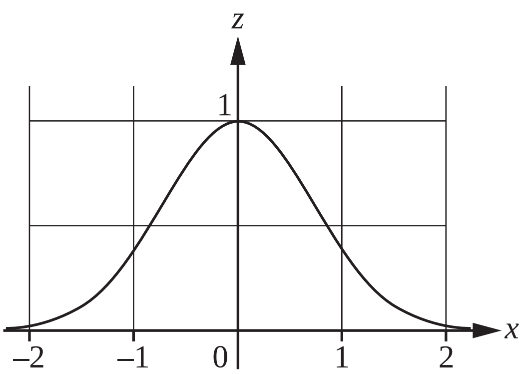

Fig. 5.1

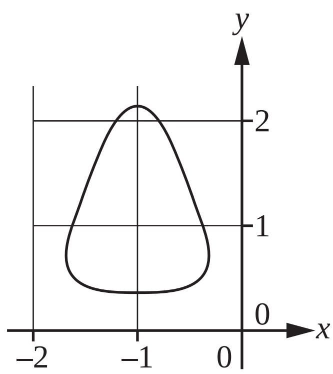

Fig. 5.2

(b) Specify a sequence of transformations which transforms the graph of $z = \mathrm{e}^{-x^2}$ onto the graph of the section defined by $z = \mathrm{f}(x, 1)$. **[2]**

(c) Hence, or otherwise, sketch the section defined by $z = \mathrm{f}(x, 1)$. **[1]**

(d) Using **Fig. 5.2** and your answer to part **(c)**, classify any stationary points on S, justifying your answer. **[2]**

You are given that P is a point on S where $z = 0$.

(e) Find, in vector form, the equation of the tangent plane to S at P. **[4]**

The tangent plane found in part **(e)** intersects S in a straight line, L.

(f) Write down, in vector form, the equation of L. **[1]**

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